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ABSTRACT

Zonally-averaged autocovariances of the five-day height tendency for lags 0, 5, ..., 25 days are computed at 25 mb around latitudes 55N, 65N, and 75N during the period 5 October 1958 through 29 March 1959. Using the convolution theorem for each space-wave number, it is possible to determine the zonally-averaged spectral density of the height-tendency field at each of these latitudes in terms of time waves having periods from 10 to 50 days. The results indicate that all but about 10% of the variance of the height-change field is accounted for by space waves one and two. For space wave number two, time waves number 3 and 4 (per 50 days) give a significant contribution to the spectrum at the 90% confidence level. However, for space wave number one, time wave number one gives the most significant contribution to the spectrum. It is inferred from this, that cases of "explosive" warming, which may occur as early as January are correlated with intensification and/or movement associated with space wave number one, at periods of approximately 100 days.

The author wishes to express his sincere appreciation to Professor Frank L. Martin, Department of Meteorology, U. S. Naval Postgraduate School, for his suggestion of the topic and his encouragement in the preparation of this study.

TABLE OF CONTENTS

LIST OF TABLES AND FIGURES

TABLE OF SYMBOLS

Symbol	Definition
Z	height of 25-mb surface in geopotential feet
$\bar{Z}$	zonal mean contour height around a latitude circle
Z^* or $Z-\bar{Z}$	perturbation height field around a latitude circle
C_n	amplitude term of Fourier Analysis for nth space harmonic wave
n	number of space waves around a latitude circle
λ	longitude
ϕ_n	phase angle of nth space harmonic wave
$\dot{Z}$	local change of 25-mb height for five days
τ	time lag
T_{max}	fundamental period
Δt	increment of time
$R(K)$	zonally-averaged time autocovariance for time lag of K units
K	lag in units of five-day periods
$R_n(K)$	zonally-averaged time autocovariance for time lag of K units and for space-wave n
η	total number of sample, in this case $\eta = 35$
$\bar{Z}$	zonal mean contour height at lag τ around a latitude circle
$\mathcal{C}_n$	amplitude of nth space wave at time t $+\tau$
ψ_n	phase angle of nth space wave at time t $+\tau$
M	maximum number of lags used in autocovariance
N	number of time waves within a fundamental time period
S_N	raw spectral density value for time wave N
$\bar{S}_N$	smoothed spectral density value for time wave N
f_N	frequency in cycles $(\text{day})^{-1}$ associated with time wave N

1. Introduction

During the past ten years, meteorologists have become aware of the winter-time phenomenon known as "explosive" warming, which occurs in the Arctic and sub-Arctic stratosphere. During this same decade, the increased availability of higher-level soundings has made accessible to the meteorologist sufficient information to analyze the structure and causes of these unusual occurrences.

The work of the McGill University group, primarily under F. K. Hare [1960] and B. W. Boville [1960] has made it clear that there is a <u>succession</u> of cold troughs and warm ridges at the 25-mb level which eventually culminates in the development of an intense warm anticyclone in polar latitudes. Northern Canada is a preferred location for the occurrence of this phenomenon. This intense anticyclogenesis may be explained [Boville, 1960] in terms of baroclinic instability in the polar night westerlies. Actually the development of the intense anticyclone represents a stage in the weakening of the polar night westerlies [Julian, 1959] which are replaced in spring and summer by the easterly wind regime that finally envelops the entire summer stratosphere. Boville and his associates [1961] have prepared an atlas of the 25-mb circulation for the six-month period 5 October 1958 through 29 March 1959. In these studies, Boville et al. list the Fourier analyses of the 25-mb contour field at five-day intervals for latitudes 55N, 65N, and 75N. For each of these latitude circles, they describe the Fourier analyses in the form

$$Z = \bar{Z} + \sum_{n=1}^{9} C_n \cos(n\lambda - \phi_n) \tag{1}$$

where the values of C_n, and ϕ_n are listed in the atlas through space-wave number nine. Here $\bar{Z}$ is the zonal mean contour height around the latitude circle while C_n and ϕ_n are the amplitude and phase angle, respectively,

of the <u>n</u>th harmonic (n=1,.....,9).

Spectral analysis has become a useful diagnostic tool of meteorologists, primarily for the purpose of detecting preferred periodicities in the eddies or perturbations under consideration. The number and variety of such studies in certain areas of statistical meteorology are too great to attempt to describe them here. Suffice it to say that the basic methods were largely developed by J. W. Tukey, and a concise summary of their applications to Meteorology can be found in the excellent textbook by Panofsky and Brier [1958].

Actually, Boville [1960] has published the spectral analysis of the 25-mb contour height field at latitude 60N for the month of October 1958. In that analysis, one of the conclusions reached was that most of the variance in the contour height field at 25 mb is explained by space-wave numbers one and two. A similar conclusion is reached in this paper in regard to the variance of the height tendency.

The objective of the present paper may be understood from the local-time differentiation of equation (1)

$$\dot{z}^* = \sum_{n=1}^{9} \left[\dot{C}_n \cos(n\lambda - \phi_n) + C_n \dot{\phi}_n \sin(n\lambda - \phi_n) \right] \qquad (2)$$

where $\dot{z}^* = \partial z/\partial t - \partial \bar{z}/\partial t$. From equation (2), it may be seen that time-changes in the perturbation field may be ascribed to terms involving $\dot{C}_n$, the <u>local-time</u> change in the harmonic wave amplitude, and $\dot{\phi}_n$, the local-time change of the phase angle. These two types of terms in equation (2) may be described as the <u>intensification</u> and <u>movement</u> contributions, respectively, for the <u>n</u>th wave. A time-spectral analysis of $\partial z/\partial t$, zonally averaged with respect to longitude, was then performed using the five-day rates of change $\dot{C}_n$ and $\dot{\phi}_n$. (For the mathematical development see section 2.) An advantage of the spectral analysis of the

height-tendency field over that of the height field alone is that the former can indicate periodicities due to <u>intensification</u> and to <u>movement</u> of the harmonic waves, whereas these connections are not so clear in the latter case.

2. Mathematical development

The autocovariance of $\dot{z}^*$ at lag τ is given [Blackman and Tukey, 1958] by the formula

$$R(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \dot{z}^*(t)\ \dot{z}^*(t+\tau)\,dt \tag{3}$$

where z^* is the zonal height perturbation, and includes the contribution from all space waves around the latitude circle, T is the averaging period over which the autocovariance is determined, τ is the lag, and t is an interval of time, also measured in five-day units. Using the truncated time series available for this study, the time integration may be replaced by a summation over η-K equally-spaced time intervals, at lag $\tau = K\Delta t$. Then the integral (3) may be rewritten in the form

$$R(K) = \frac{1}{2\pi}\int_0^{2\pi} \frac{1}{\eta - K}\sum_{t=1}^{\eta-K}\left[\dot{z}^*(t)\cdot \dot{z}^*(t+\tau)\right]\delta\lambda, \qquad 0 \leq K \leq M, \tag{4}$$

where R(K) is the zonally-averaged time autocovariance. In equation (4), λ represents the longitude at any point within a latitude circle. Here, K is the number of lags in terms of intervals of Δt = 5 days, η is the number of five-day height changes available in the time series, and M is the maximum number of time lags used in this study. Obtaining the five-day perturbation height tendency $\dot{z}^*(t+\tau)$ at time $t+\tau$ by an equation analogous to (2), we may write,

$$\dot{z}^*(t+\tau) = \sum_{n=1}^{9}\left[\dot{C}_n \cos(n\lambda - \psi_n) + C_n\dot{\psi}_n \sin(n\lambda - \psi_n)\right], \tag{5}$$

where C_n and ψ_n are analogous to C_n and ϕ_n of equation (1). Hence, from equations (2), (4), and (5), the zonally-averaged time autocovariance for space wave n has the form

$$R_n(K) = \frac{1}{n-K} \sum_{t=1}^{n-K} \left\{ \left[\frac{\dot{C}_n \dot{\mathcal{C}}_n + C_n \mathcal{C}_n \dot{\Phi}_n \dot{\Psi}_n}{2} \right] \cos(\Psi_n - \Phi_n) + \right.$$

$$\left. \left[\frac{\dot{\mathcal{C}}_n C_n \dot{\Phi}_n - \mathcal{C}_n \dot{C}_n \dot{\Psi}_n}{2} \right] \sin(\Psi_n - \Phi_n) \right\}. \qquad (6)$$

In equation (6) the subscript n indicates that $R_n(K)$ is independent of all space waves other than the <u>n</u>th. In the derivation of (6), considerable simplification has been made possible as a result of orthogonality properties, that is by deleting terms which vanish upon integration around a complete latitude circle. A proof of equation (6) is given Appendix I. Since discrete data and a finite time series have been used, the maximum lag $M\Delta t$ limits the maximum resolvable period to be $T_{max} = 2M\Delta t$, which may be called the <u>fundamental period</u>. In this study the maximum number of lags has been taken as M = 5 so that the fundamental period will be 50 days. If K is allowed to range over the values 0, 1, 2, ..., M, various values of $R_n(K)$ can be generated for the different space-wave numbers. Blackman and Tukey [1958] recommend, when using a truncated time series, that the maximum number of lags M should not exceed approximately 10% of the length of the record. Since the total available number of five-day mean values of $\partial z/\partial t$ was 35, the maximum number of lags was taken to be five. This lag number was considered to be in reasonable accord with the recommendation of Blackman and Tukey, and yet sufficiently large to give resolution for the long-period waves which occur in this meteorological regime.

The definition of the spectral density at time-wave number N [Kahn, 1957] is the average contribution to the variance from a unit wave number interval centered at wave number N. Since the maximum period under consideration is 50 days, the concept of a time-wave number N is to be inter-

preted in terms of frequencies as follows:

1 wave per 50 days for N = 1,

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N waves per 50 days for wave number N, $N \leq 5$.

Thus the frequency f_N may be expressed as

$$f_N = \frac{N}{2M\Delta t}, \quad N \leq M, \tag{7}$$

and spans the frequency-interval $(N - \frac{1}{2})/2M\Delta t$ to $(N + \frac{1}{2})/2M\Delta t$. The corresponding period for this time-wave is the reciprocal of f_N. The highest frequency f_F resolvable when using such discrete data at finite intervals occurs for N = M and is

$$f_F = \frac{1}{2\Delta t}$$

f_F is often called the <u>folding</u> or <u>Nyquist</u> frequency, and in the present study is one cycle $(10 \text{ days})^{-1}$.

Panofsky and Brier [1958] give the following equation for raw spectral density estimates for time-wave number N:

$$S_N = \frac{2}{M}\left\{\frac{1}{2}\left[R(0) + (-1)^N R(M)\right] + \sum_{K=1}^{M-1}\left[R(K) \cos\frac{\pi N K}{M}\right]\right\}, 1 \leq N \leq M-1. \tag{8}$$

In practice, S_N is generally considered to be distributed uniformly over the unit wave-number interval $N - \frac{1}{2}$ to $N + \frac{1}{2}$, except at N = 0 and N = M.

For the special cases N = 0 and N = M, the corresponding estimates S_0 and S_M follow from equation (8) multipled by one-half because only one-half of a wave-number interval is available at N = 0 and N = M. These unsmoothed raw spectral estimates may be smoothed by "hanning" [Blackman and Tukey, 1958], in the following manner

$$\overline{S}_N = .25\, S_{N-1} + .5\, S_N + .25\, S_{N+1}, \tag{9}$$

except at the extreme wave numbers N = 0 and N = M where the average is modified to

$$\bar{S}_0 = .5 S_0 + .5 S_1 , \quad \bar{S}_M = .5 S_{M-1} + .5 S_M. \tag{10}$$

These smoothed estimates may be considered to be centered at wave number N, or frequency f_N given by equation (7).

Note that the integrated value of all the spectral estimates for a particular space-wave n gives the total time variance, for this wave. In other words, by definition, the relationship

$$\sigma_n^2 = \sum_{N=0}^{5} S_{N,n} \Delta N \tag{11}$$

holds, although carrying the integration only to N = 5 neglects some of the higher frequency spectral contributions. In addition, "aliasing" or folding of high frequency energy back upon the Nyquist frequency has been neglected. However the spectral densities at higher space-wave numbers (table 2) suggest that this is not important in the present problem. This has been inferred since the contributions to the time variance for these wave numbers appears to act as "noise", with no preference for the higher time-wave numbers.

3. Computational procedures and interpretations

As mentioned in section 2, the maximum number of time lags chosen was M = 5. This choice gives a resolution of the spectrum of $\partial z/\partial t$ to a maximum frequency of 1/10 cycle $(\mathrm{day})^{-1}$.

All computational processes were performed on the CDC-1604 computer. All programs devised for this study were written by the author or modified from those available in the U. S. Naval Postgraduate School computer center library. The three latitudes chosen for analysis were 55N, 65N, and 75N. Table 1, a, b, c gives the zonally-averaged autocovariances for each space-wave number computed by equation (6) at each of the three latitude circles. The smoothed density spectra for each space-wave number were then computed according to equations (8), (9), and (10) and the results displayed in table 2, a, b, c.

Note that in table 2, a, b, c periodicities are presented down the left-hand column, while the space-wave numbers n = 1, ..., 9 appear along the top of the table. From the rows denoted "P. R. Var.", it may be seen that the space waves n = 1 and n = 2 together account for approximately 90% of the variance of the zonally-averaged five-day height changes. While the percent reduction of the variance of $\partial z/\partial t$ accounted for by space wave 2 exceeds that of wave 1, the contribution of the latter increases as one proceeds from latitude 55N to 75N. The contribution by waves 3 and 4 accounts for less than 10% at latitudes 55N and 65N, and for less than 5% at latitude 75N. Thus, in this study, only the time-spectra of space waves one and two have been analyzed for significant periodicities.

For space waves n = 1 and n = 2, fiducial confidence limits at the 10% and 90% significance level have been taken from Blackman and Tukey [1958]. These limits are based upon the χ^2/f distribution, where

[illegible]

[illegible]

[illegible]

(a) 55N

LAG DAYS	SPACE WAVE NUMBERS 1	2	3	4	5	6	7	8	9
0	42.18	218.66	11.80	13.43	5.35	1.64	2.01	2.04	.74
5	8.86	-31.97	2.63	-1.28	.31	.14	.01	.14	.04
10	2.43	-31.53	4.22	.75	.26	-.02	.16	.19	.04
15	-3.37	6.46	1.71	2.55	.79	-.06	.32	.45	.04
20	.86	5.03	3.25	.44	-.72	.14	.40	.37	-.03
25	-8.46	-11.47	2.22	1.29	.36	.30	.32	.54	.14

(b) 65N

0	80.78	336.38	15.31	27.36	3.11	.97	1.37	1.14	.27
5	23.62	-139.61	1.25	5.23	.53	.12	-.01	.07	.01
10	-1.55	25.27	-2.33	8.17	.07	-.02	.58	.09	-.02
15	-4.05	14.77	.50	5.78	.04	-.01	.02	.18	-.02
20	1.33	-13.28	.58	-.28	.12	-.03	.23	.24	.07
25	-17.53	-11.36	1.18	3.94	-.10	.00	.08	.11	.03

(c) 75N

0	103.59	139.47	6.21	2.69	1.39	1.07	.34	.36	.23
5	28.08	-31.36	.97	.44	-.04	.22	.02	.10	.04
10	7.89	4.73	-1.02	.17	-.01	-.03	-.00	.03	.06
15	-11.79	-1.13	-.10	.79	.06	.09	.03	.02	.02
20	-4.05	42.43	.19	-.23	.25	.05	.11	.04	-.03
25	-11.88	-46.88	.41	-.26	.01	.20	.09	.04	-.00

Table 1. Zonally averaged autocovariances of $\partial z/\partial t$ for lags of 0, 5, ..., 25 days at latitudes (a) 55N; (b) 65N; (c) 75N. The units are in terms of 10^4 (feet/5 days)^2.

(a) 55N

FREQUENCY CYCLES/DAY	SPACE WAVE NUMBERS								
	1	2	3	4	5	6	7	8	9
0/50	9.28	19.34	2.89	1.81	.98	.25	.30	.34	.13
1/50	10.06	28.31	2.36	1.76	1.00	.31	.29	.31	.14
2/50	9.30	46.17	1.61	2.11	.95	.36	.34	.34	.14
3/50	6.57	54.77	1.40	2.96	1.06	.31	.41	.41	.14
4/50	4.71	44.96	1.51	2.92	1.01	.24	.36	.36	.13
5/50	4.05	34.94	1.61	2.41	.79	.21	.30	.29	.12
P. R. Var.	14.2	77.0	3.5	4.5	1.9	0.7	0.6	0.7	0.3

(b) 65N

0/50	18.69	19.95	2.17	6.91	.62	.18	.28	.19	.03
1/50	20.53	25.75	2.79	5.78	.67	.20	.23	.18	.04
2/50	19.59	44.51	3.65	3.78	.66	.21	.15	.20	.06
3/50	13.41	79.03	3.48	3.90	.55	.18	.16	.22	.06
4/50	7.44	95.69	2.33	4.52	.41	.14	.25	.19	.04
5/50	4.88	90.79	1.60	4.13	.33	.11	.29	.15	.03
P. R. Var.	16.0	75.0	3.1	5.2	0.6	0.2	0.2	0.2	0.1

(c) 75N

0/50	24.56	13.15	.98	.60	.18	.21	.05	.08	.06
1/50	26.90	14.79	1.27	.53	.21	.22	.05	.08	.05
2/50	23.46	24.01	1.58	.46	.27	.24	.07	.08	.03
3/50	14.54	30.77	1.34	.54	.29	.21	.07	.06	.03
4/50	10.50	27.46	.81	.46	.24	.14	.05	.04	.04
5/50	9.57	24.97	.53	.29	.20	.10	.04	.03	.03
P. R. Var.	42.0	53.0	2.6	1.1	0.5	.4	0.1	0.1	0.1

Table 2. Zonally-averaged spectral densities for space-wave number n=1, ...,9, and wave frequency N=0,1/50,...,5/50 cycles/day. Sections (a), (b), and (c) are for latitudes 55N, 65N, and 75N. The units are in terms of 10^4(feet/5 days)^2.

$f = (2\mathcal{N} - M/2)/M$ is the number of degrees of freedom. In this investigation we have $\mathcal{N} = 35$ and $M = 5$, so that $f \approx 14$. For these values, the 10% and 90% fiducial limits interpolated from those given by Blackman and Tukey [1958, p.208] are 0.55 and 1.51 respectively.

Figure 1 illustrates the time-spectral analysis at 65N, for space waves n = 1 and n = 2 (the latter shown by the solid line). From this diagram it may be seen that for n = 2, the mean variance averaged over all periods is $60.1 \times 10^4 (\text{ft}/5 \text{ days})^2$. The fiducial limits have been constructed relative to this mean. Thus it is evident that for n = 2, waves of period 10 days and 12.5 days give significantly large contributions at a 90% confidence level. On the other hand, at 65N space wave 2 at a period of 50 days, gives significantly small contributions to the variance.

The same type of analysis for n = 1 at latitude 65N shows that the mean variance for space wave n = 1, averaged over all periods is 13.5, in units of $10^4 (\text{ft}/5 \text{ days})^2$, so that a time-wave of period 50 days gives a significant contribution at a 90% confidence level. It should be recalled that a period of 50 days in figure 1 actually spans a range of periods from 33.3 to 100 days, and such waves could include those associated with explosive warming. This is so, since strictly speaking, the largest value of $\partial C_1/\partial t$ which contributes to the variance of $\partial z/\partial t$ could occur at a time which is either three-fourths or one and three-fourths of a period after the onset of the autumnal cooling season in polar latitudes.

In table 3 the mean spectral densities at latitudes 55N and 75N, averaged over all frequencies in table 2, are given for space waves n = 1 and n = 2.

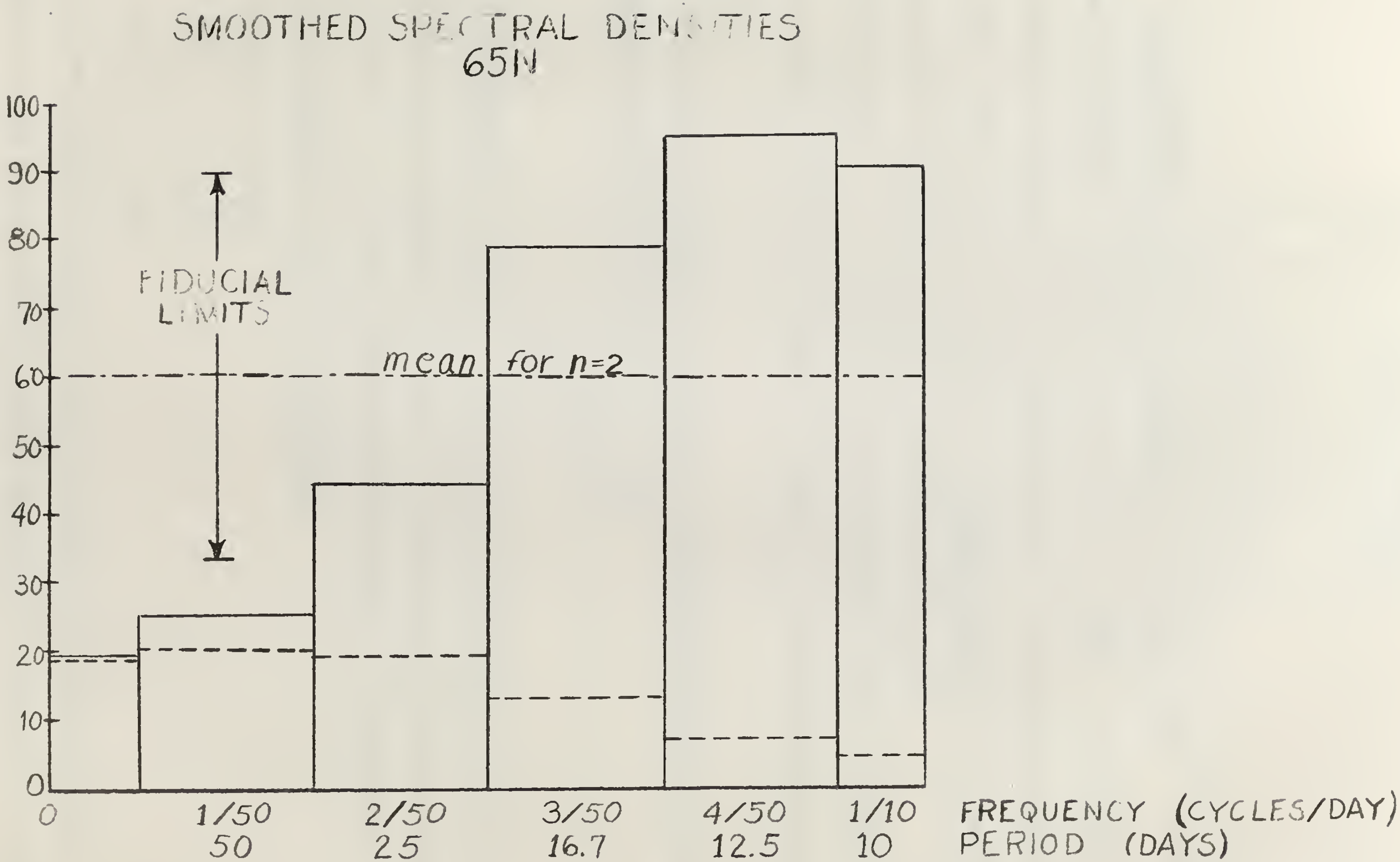

Figure 1. Smoothed time spectral densities in units of $10^4(\text{feet}/5\text{ days})^2$, at 65N. The solid line is for space wave n = 2, while the dashed line is for space wave n = 1.

LAT	SPACE - WAVE	
	1	2
55	7.4	40.4
75	18.5	23.2

Table 3. Mean spectral densities in units of 10^4(feet/5 days)^2 at latitudes 55N and 75N for space-waves n = 1 and n = 2.

Using the fiducial confidence limits for the χ^2/f distribution previously mentioned (0.55 and 1.51, respectively), it is apparent from table 2 (a, c) that there exists no significant preferred peak in the spectral densities at the 90% limit for either latitude 55N or 75N.

For space wave n = 1, at each of the latitude circles of table 2, there is a steady decrease of spectral density, S_N, with increasing time-wave $N \geq 1$.

For the case of isotropic turbulence, MacCready [1953] has tested the applicability of a spectral function of form

$$S_N \propto N^{-p}$$

where N is the time-wave number. This test is essentially based upon a theory of isotropic turbulence set forth by Heisenberg [1948] in which p = 5/3. For n = 1, and using the results of table 2, computation of the values of the exponent p gave the following:

55N, p = 0.57

65N, p = 0.91

75N, p = 0.64

Chiu [1960] has obtained somewhat similar results for the exponent p using the spectrum of the zonal and meridional components of the wind at Belmar and Cocoa, and finds values of p of the order of 1.

On the other hand, the results of this study for space wave n = 2 do

not show the decrease of spectral density with increasing time-wave number. Hence for $n = 2$, the peak spectral density which occurs in each case of table 2 at either time-wave 3 or 4 is even more likely to be significant of the average state of synoptic events.

Space-wave $n = 1$, with its maximum spectral contribution at time-wave $N = 1$ is capable of explaining those perturbations of longer period associated with "explosive" warming.

4. Conclusions

(1) Space waves 1 and 2 contribute 90% of the variance of the height-tendency field at latitudes 55N, 65N, and 75N.

(2) The chief contributors to the spectrum of $\partial z/\partial t$ occur for time-waves N = 3 or N = 4 (periods 12.5 and 16.7 days), which have the same general periodicity as the average perturbation period at a fixed station [see, for example, Austin and Krawitz, 1956].

(3) The chief contributor to the time variance for <u>space wave</u> n = 1 occurs for time wave N = 1, or possibly N = $\frac{1}{2}$, which corresponds to periods of the order of 50 days or longer. Such periods may be sufficiently long to account for the "explosive" warming which sometimes occurs as early as January.

(4) For space wave n = 1, the time spectral density was distributed with time wave number according to

$$S_N \propto N^{-0.71}$$

which compares reasonably with the results obtained in other spectral studies [c.f. Chiu, 1960].

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APPENDIX I

DERIVATION OF THE ZONALLY-AVERAGED TIME AUTOCOVARIANCE

The equation for the zonally-averaged time autocovariance of the height-tendency field at lag τ may be expressed in the form shown in equation (4)

$$R(K) = \frac{1}{2\pi}\int_0^{2\pi} \frac{1}{\eta-K}\sum_{t=1}^{\eta-K}\left[\dot{z}^*(t) \quad \dot{z}^*(t+\tau)\right]\delta\lambda, \tag{4}$$

where η is the maximum number of discrete values in the sample, and $R(K)$ is the zonally-averaged time autocovariance.

When equation (4) is expanded using equations (2) and (5), and upon interchanging the order of summation, equation (12) is obtained

$$R_n(K) = \frac{1}{\eta-K}\sum_{t=1}^{\eta-K}\frac{1}{2\pi}\int_0^{2\pi}\left\{\frac{\dot{C}_n\dot{C}_n}{2}\left[\cos(2n\lambda-\phi_n-\psi_n)+\cos(\psi_n-\phi_n)\right]+\right.$$

$$\frac{C_n\dot{\Phi}_n\dot{C}_n}{2}\left[\sin(2n\lambda-\phi_n-\psi_n)+\sin(\psi_n-\phi_n)\right]+$$

$$\frac{C_n\dot{\Psi}_n\dot{C}_n}{2}\left[\sin(2n\lambda-\phi_n-\psi_n)-\sin(\psi_n-\phi_n)\right]+$$

$$\left.\left.\frac{C_n\dot{C}_n\dot{\Psi}_n}{2}\left[\cos(\psi_n-\phi_n)-\cos(2n\lambda-\phi_n-\psi_n)\right]\right\}\delta\lambda\ \right], \tag{12}$$

where the orthogonality property has also been used. Equation (12) represents a double integral; first, with respect to longitude, second, with respect to time. The integral with respect to longitude gives the zonal average. On integrating around the latitude circle from 0 to 2π, all cosine and sine terms involving the argument $(2n\lambda-\phi_n-\psi_n)$ integrate to zero. Therefore, the result obtained is

[illegible]

$$R(K) = \sum_{n=1} \frac{1}{N-K} \sum_{t=1}^{N-K} \left\{ \frac{\left[\dot{C}_n \mathring{\varphi}_n + C_n \varphi_n \dot{\Phi}_n \mathring{\Psi}_n \right]}{2} \cos\left(\Psi_n - \Phi_n\right) + \frac{\left[C_n \mathring{\varphi}_n \dot{\Phi}_n - \varphi_n \dot{C}_n \dot{\Psi}_n \right]}{2} \sin\left(\Psi_n - \Phi_n\right) \right\}$$

$$= \sum R_n(K) \, . \qquad (6)$$

Note that $R_n(K)$ is the zonally-averaged autocovariance at a time-lag of $K\Delta t$ days, for the nth space-wave; no other space-wave appears in the expression for $R_n(K)$. The interval between successive data is such that $\Delta t = 5$ days.

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